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A Note on the Application of the Extended Bernoulli Equation

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A Note on the Application of the Extended Bernoulli Equation

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Abstract

A general form of the momentum equation is presented. Because the solution is presented as an integral along a flow line, it is here referred to as an "extended" Bernoulli equation. The equation, as presented, is valid for unsteady, compressible, rotational, elasto-viscoplastic flows measured relative to a noninertial (translationally and/or rotationally accelerating) coordinate system, whose motion is known. Though all of these concepts have long been separately addressed in the educational literature of fluid and solid mechanics and dynamics, they are usually not available from a single source, as the literature prefers to reduce the problem to special-case solutions for instructional purposes. Two examples that make use of the extended Bernoulli equation in noninertial reference frames are solved. The consequences of failing to properly account for noninertial effects are discussed.

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1. Introduction

The Bernoulli equation is perhaps the most famous and widely used equation in fluid mechanics, relating the pressure, p, in a flow to the local velocity, V, and gravity potential. It was derived by considering a balance of momentum along a streamline, for the special case of steady, incompressible, inviscid flow in an inertial reference frame, with gravity as the only significant body force. The Bernoulli equation may also be derived from considerations of energy conservation, since, for inviscid flows, there is no energy loss. The Bernoulli equation is given as

$$\rho \frac{V^2}{2} + p + \rho g h = \text{constant} \quad , \tag{1}$$

where ρ is the flow density, g is the acceleration due to gravity, and h is the vertical height of the flow relative to some reference location. Furthermore, if the flow is irrotational, the constant of eqn (1) will be the same for all streamlines throughout the flow. The real world is rarely so kind as to satisfy all the restrictive conditions under which eqn (1) was derived. Yet, because the influence of these nonideal (compressible, viscous, rotational, accelerational) terms is often small, the engineering world makes great use of eqn (1), often modifying it in an *ad hoc* manner when nonideal effects rear their ugly head.

We endeavor here to pull together various equations and constructs from the literature into a single framework, to present an unsteady, compressible, rotational, elasto-viscoplastic, noninertially referenced momentum equation with no presuppositions. The importance of each term can then be examined at the time of application to ascertain when discarding or approximating it is appropriate. Because our primary interest in the subject lies in the area of noninertial coordinate systems, examples of this variety, which make use of what might be called an extended Bernoulli equation, are presented.

All of the concepts relating to the momentum equation that are discussed in this report are readily available in one form or another throughout the educational literature of fluid mechanics, solid mechanics, and dynamics. They are, however, not always found in a single

location. Furthermore, in an effort to teach textbook examples and solve textbook problems, the educational literature quickly reduces the governing equation to certain well-known academic cases, often failing to give full coverage to the general case involving viscous (or rotational), compressible, accelerating, nonsteady flows in noninertial coordinate systems.

For example, Shames [1] generally does an excellent job of covering most aspects of the momentum equation and noninertial reference frames, though it is done in terms of finite-sized control volumes and not streamline-sized "flow tubes." Potter and Foss [2] cover all the relevant equations regarding the forces and accelerations upon a material point in a flow, but fail to tie the equations together into a generalized unsteady Bernoulli equation. Kelley [3] derives an extended Bernoulli equation, but only for the case of nonviscous flows in inertial coordinate systems. Currie [4] also derives a restrictive form of the extended Bernoulli equation, valid only for irrotational flow in an inertial reference frame. The very thorough Schlichting [5], because of its emphasis on boundary layers, does not even address the issue of noninertial coordinate systems. Greenspan [6] examines the momentum equation in a noninertial frame, but only for the special case of purely rotating frames, as might be found in the case of rotating fluid problems. In addition to addressing the steady-state Bernoulli equation for streamlines, Lamb [7], like Shames [1], also covers aspects of noninertial frames, but on an integrated volume basis. Thus, this report is intended merely to serve as a handy repository of several important well-established concepts that might otherwise need to be tracked down in a multiplicity of texts and chapters.

2. The Momentum Equation and Special Cases

The momentum equation on a continuum element of material, which can be found in many texts (*e.g.*, Potter and Foss [2]), is given as

$$\frac{DV}{Dt} = \frac{s_{ij,j} - \nabla p}{\rho} + \nabla\Phi \ , \tag{2}$$

where D/Dt denotes the material derivative (discussed in following section); V is the vector velocity of the material element, as measured in an inertial reference frame; p is the element

pressure; ρ is the element density; Φ is the body force potential; ∇ is the vector gradient operator; s_{ij} is the deviatoric-stress tensor arising from any type of elasto-viscoplastic constitutive behavior; and $s_{ij,j}$ is index notation for $\partial s_{ij}/\partial x_j$, denoting the following vector condensation of the deviatoric-stress tensor:

$$s_{ij,j} = \left(\frac{\partial s_{xx}}{\partial x} + \frac{\partial s_{xy}}{dy} + \frac{\partial s_{xz}}{\partial z}\right)\hat{i} + \left(\frac{\partial s_{yx}}{\partial x} + \frac{\partial s_{yy}}{\partial y} + \frac{\partial s_{yz}}{\partial z}\right)\hat{j} + \left(\frac{\partial s_{zx}}{\partial x} + \frac{\partial s_{zy}}{\partial y} + \frac{\partial s_{zz}}{\partial z}\right)\hat{k} \; . \tag{3}$$

Eqn (2) is a general form of the momentum equation from which many commonly employed special cases derive.

For example, when deviatoric stresses, s_{ij}, are zero, as in the case of an inviscid fluid, the momentum equation, eqn (2), becomes the well-known Euler's equation,

$$\frac{DV}{Dt} = -\frac{\nabla p}{\rho} + \nabla\Phi \; . \tag{4}$$

For a body in equilibrium, where the material accleration, DV/Dt, is everywhere zero, but where deviatoric stresses, s_{ij}, may arise from elastic strains in the body, eqn (2) reduces to the equilibrium equation of solid mechanics,

$$\sigma_{ij,j} + \rho\,\nabla\Phi = 0 \;, \tag{5}$$

where $\sigma_{ij,j}$ is the absolute-stress tensor condensation resulting from the combination of the pressure gradient and deviatoric-stress condensation. On the other hand, if the flow is accelerational, but the deviatoric stresses in eqn (2) arise solely from Newtonian viscosity, μ, in which shear stress is proportional to the associated velocity gradients (and assuming the validity of Stokes' hypothesis), then the deviatoric-stress condensation can be expressed in terms of velocity gradients (*e.g.*, Schlichting [5]) to give the famous Navier-Stokes equation,

$$\rho\frac{DV}{Dt} = \rho\,\nabla\Phi - \nabla p + \mu\,\nabla^2 V + \frac{\mu}{3}\nabla(\nabla\cdot V) \; . \tag{6}$$

For incompressible viscous flow, the last term of eqn (6) will vanish, since, for incompressible flow, the divergence of velocity is identically zero. This incompressible form of eqn (6) is known

as the incompressible Navier-Stokes equation. Eqns (4)–(6) each represent a useful special-case solution of the general momentum equation, eqn (2).

3. Lagrangian vs. Eulerian Acceleration

Focusing on the left side of eqn (2), the *material* (also known as *total* or *substantive*) *acceleration*, DV/Dt, denotes the acceleration experienced by "any one" particle of material as it traverses the flow field. In essence, it is the acceleration that would be measured by an infinitesimal accelerometer immersed in and traveling with the surrounding flow. The acceleration, DV/Dt, is associated with a Lagrangian description of the flow field, in which $V = V(x,y,z,t)$. In the Lagrangian description, x, y, and z are variables that, when taken as spatial coordinates (x,y,z), define a particular material particle present at that coordinate at some given reference time, t_0. Once a material particle is defined (*i.e.*, once the variables x, y, z are fixed to particular values), the behavior of that particle becomes a function of time only and derivatives with respect to time (*e.g.*, acceleration) describe the time rate of change as perceived by the material particle in question. The D/Dt operator denotes these Lagrangian temporal derivatives, for the special case where the particular material point (x,y,z) is defined when the reference time, t_0, is set to the current time, t, such that $DV/Dt = d/dt(V[x(t),y(t),z(t),t])$.

Often, however, it is (mathematically or experimentally) more convenient to measure flow properties (like acceleration) at fixed locations in space, rather than moving with a material particle. This framework is associated with the Eulerian description of the flow field, in which $V = V(x,y,z,t)$. Unlike the Lagrangian description, however, in which the coordinates (x,y,z) define a material particle at some reference time, t_0, the Eulerian variables x, y, and z define points that are forever fixed in coordinate space, even as material flows through that space. The measure of flow acceleration in this description, referred to as the *local acceleration*, is performed at a fixed point in space and denoted $\partial V/\partial t$ since spatial coordinates x, y, and z are held constant when computing the time rate of change. Lumley [8] provides an excellent comparison of these two frameworks. All undergraduate fluid mechanics texts derive the equations interrelating these two frameworks, which are simply presented here as

$$\frac{d}{dt}V[x(t),y(t),z(t),t] = \frac{DV}{Dt} = \frac{\partial V}{\partial t} + (V\cdot\nabla)V \ . \tag{7}$$

In addition to the local acceleration, it is seen that the material acceleration is composed of terms known as *acceleration of transport*, or *convective acceleration*, given by the last term of eqn (7). This equation reveals how conditions in a flow field at all points fixed in space can be steady ($\partial V/\partial t = 0$), while, at the same time, any material element of that flow experiences all manner of accelerations as it traverses the field ($DV/Dt \neq 0$).

Furthermore, a number of texts (*e.g.*, Potter and Foss [2]) also present a form of eqn (7) that has been manipulated via vector mechanics, to yield

$$\frac{DV}{Dt} = \frac{\partial V}{\partial t} + \nabla\left(\frac{V^2}{2}\right) + (\nabla\times V)\times V \ . \tag{8}$$

This form is especially interesting because it separates the V^2 inertial-force term from the vorticity-induced term involving cross products. For flows that are irrotational, all terms involving vorticity, $\nabla\times V$, will vanish. Furthermore, the inertial-force term is the genesis of the V^2 dependence of the Bernoulli equation, eqn (1).

4. Noninertial Reference Frames

In the momentum equation, eqn (2), the material acceleration must be measured with respect to an inertial reference frame. However, both experimentally and analytically (*e.g.*, as in the case of potential flow), it is often more convenient to measure coordinates with respect to a body of interest within the flow field. If the body moves with constant velocity, then such body coordinates serve also as an inertial reference system. If, however, the forces of the flow upon the body serve to accelerate the body, the body coordinates are no longer inertial and eqn (2) is no longer valid as measured in the body coordinate frame.

Any undergraduate dynamics text (*e.g.*, Beer and Johnson [9]) and many fluid mechanics texts (*e.g.*, Shames [1] and Potter and Foss [2]) derive or present the equation for acceleration of a particle, when the kinematics of particle motion are measured with respect to a noninertial

reference frame xyz. Using the notation of Figure 1, in which the noninertial frame xyz moves with respect to an inertial reference frame XYZ by way of translation vector $S(t)$ and rotation vector $\Omega(t)$, while the kinematics of the particle motion in question are measured with respect to xyz by the displacement vector $R(t)$, one obtains

$$A = a + \frac{d^2S}{dt^2} + 2\Omega\times V_{xyz} + \Omega\times(\Omega\times R) + \frac{d\Omega}{dt}\times R \ , \tag{9}$$

where A is the total acceleration with respect to XYZ; $V_{xyz} = dR/dt$ is the velocity measured in the noninertial xyz frame; and $a = DV_{xyz}/Dt$ is the material acceleration, as measured in xyz.

Since eqn (2), the momentum equation, can only be valid when applied in an inertial frame, eqn (9) provides the means to apply eqn (2) with respect to the inertial XYZ, even when the flow kinematics (*e.g.*, R and V) are measured with respect to a translating and rotating xyz, which are perhaps attached to a body of interest. Realizing that the inertial acceleration, A, in eqn (9) corresponds to the material acceleration, DV/Dt, presented on the left side of the inertially constrained eqn (2), we have by substitution

$$A = \frac{DV_{xyz}}{Dt} + \frac{d^2S}{dt^2} + 2\Omega\times V_{xyz} + \Omega\times(\Omega\times R) + \frac{d\Omega}{dt}\times R = \frac{s_{ij,j} - \nabla p}{\rho} + \nabla\Phi \ . \tag{10}$$

Substitution of eqn (8) for the noninertial xyz material acceleration, DV_{xyz}/Dt, and some simple rearrangement gives the following result:

$$\frac{\partial V_{xyz}}{\partial t} + \frac{d^2S}{dt^2} + (\nabla\times V_{xyz})\times V_{xyz} + 2\Omega\times V_{xyz} + \Omega\times(\Omega\times R) + \frac{d\Omega}{dt}\times R = \nabla\Phi + \frac{s_{ij,j} - \nabla p}{\rho} - \frac{\nabla V_{xyz}^2}{2} \ . \tag{11}$$

Eqn (11) is valid at all points in a compressible, elasto-viscoplastic, rotational, nonsteady flow subject to conservative body forces, as measured in a reference frame undergoing time-dependent translational and rotational motions.

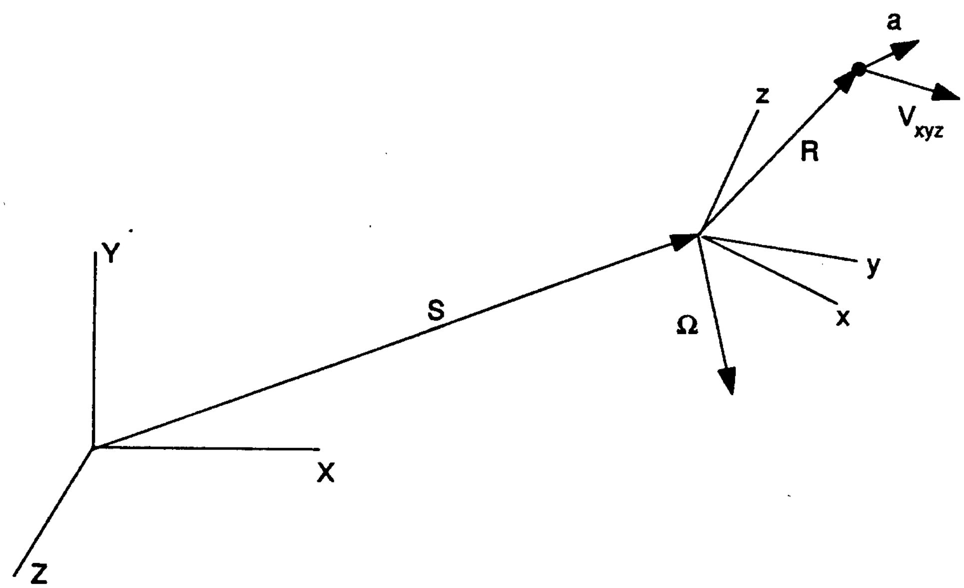

Figure 1. Depiction of the noninertial reference frame *xyz* translating (*S*) and rotating (Ω) with respect to inertial frame *XYZ*. Flow kinematic variables *R*, V_{xyz}, and *a* are measured with respect to the noninertial *xyz* frame.

The first two terms of eqn (11) represent the inertial XYZ rigid-body translational acceleration of the material point in question. The third term, involving vorticity, $\nabla \times V$, represents acceleration resulting from flow vorticity, as measured in xyz. The remaining terms on the left-hand side represent Coriolis, centripetal, and rotational accelerations arising strictly from the time-dependent rotational motion of the noninertial reference frame xyz. On the right side of eqn (11), the traditional Bernoulli force terms (body, pressure, and inertial) as well those involving deviatoric-stress gradients are found.

5. Extended Bernoulli Equation

While the term "Bernoulli equation" describes only the relation given in eqn (1), it is popular to use the term "Bernoulli" to describe the momentum equation when integrated along a contour in a flow field, even if the restrictive conditions (steady, incompressible, inviscid flow in an inertial reference frame, with gravity as the only significant body force) that are in force for eqn (1) have been relaxed. In this spirit, eqn (11) may be integrated along an arbitrary flow contour fixed in noninertial xyz space (thus translating and rotating with S and Ω in XYZ space) and the result referred to as an extended Bernoulli equation. The contour integration yields

$$\int_{R_1}^{R_2}\left(\frac{\partial V_{xyz}}{\partial t} + \frac{d^2S}{dt^2}\right)\cdot dR + \int_{R_1}^{R_2}(\nabla\times V_{xyz})\cdot(V_{xyz}\times dR)$$

$$+ \int_{R_1}^{R_2}\left(2\Omega\times V_{xyz} + \Omega\times(\Omega\times R) + \frac{d\Omega}{dt}\times R\right)\cdot dR = (\Phi - V_{xyz}^2/2)\Big|_{R_1}^{R_2} + \int_{R_1}^{R_2}\left(\frac{s_{ij,j}}{\rho} - \frac{\nabla p}{\rho}\right)\cdot dR \quad , (12)$$

where the vector increment dR is made to follow the path of the contour throughout the integration. This result is valid for nonsteady, compressible, rotational, elasto-viscoplastic flows in a noninertial reference frame. Note that a minor vector manipulation has been performed upon the vorticity integral term. Furthermore, the gradient integrals of inertial and body forces on the right-hand side of eqn (12) were reduced to a difference in the values of $V^2/2$ and Φ between the two endpoints of the contour. The pressure gradient integral may also be a function of the contour endpoint values, (p/ρ), but only if the flow is incompressible; otherwise, the term must be integrated along the contour. Unfortunately, the deviatoric-stress integral must, in general, be

explicitly performed, as it does not represent a gradient potential. The contour of integration in eqn (12) may be any arbitrary three-dimensional contour. However, the resulting equation, because of the vector mathematics, will be scalar.

Different types of problems will employ different terms of this equation. For example, problems of fluids involving turbomachinery will allow the first integral to be discarded if the problem, when viewed in a rotating coordinate system can be made to appear steady. For irrotational flows, such as are found in many applications of potential flow theory, the second integral may be discarded. Even when the flow is rotational, if the integration contour is, at a particular instant in time, also a streamline (*i.e.*, everywhere parallel to the velocity vector), the second integral also disappears, as $V \times dR$ will be zero at all places along a streamline. For the problem of linearly accelerating bodies within a flow field, the noninertial body-coordinate system *xyz* need not rotate and the third integral may therefore be discarded for problems of this type. Typical conditions that could justify elimination of terms on the right-hand side of the equation would involve negligible body forces and/or shear stresses (inviscid, nonelastic).

Several examples involving the use of eqn (12) are now investigated. Because our primary interest in the subject lies in the area of noninertial coordinate systems, we will focus on problems of this type.

6. Nonsteady Potential Flow Around a Sphere

The use of flow potentials to solve a variety of steady flow problems is a well-established procedure in fluid mechanics textbooks. Mention is usually made of nonsteady potential flow by showing an equation involving a time-derivative of the potential, but nonsteady potential-flow problems are not typically solved or explained in textbooks. One reason becomes quickly apparent, when it is considered that most potential flow fields extend infinitely in at least one direction. In particular, any time-dependent variation of a potential flow field will often involve time-dependent variations at infinity. Time-dependent velocities involve accelerations, and accelerations require forces. And though steady flow around a fixed body is inertially equivalent

to that body's uniform motion through a quiescent medium, it is most definitely **not true** that the force required to accelerate a body through a medium at rest (*e.g.*, the universe) is identical to the force required to accelerate the universe around the at-rest body.

Fortunately, eqn (12) allows one to overcome this difficulty. An unsteady potential flow problem, in which the universe is allowed to accelerate about a fixed-in-space potential-flow body, can be solved, as long as it is realized that the potential-flow body is fixed in noninertial *xyz* space—a coordinate system that is, in effect, accelerating equal and opposite to the acceleration of the potential-flow far-field. In this way, the net far-field acceleration is zero and the unsteady motion of a body through an inertial flow field is truly modeled. This argument is valid for both linear and rotational far-field accelerations.

The potential-flow solution for invisicid, nonrotational, incompressible, uniform flow about a rigid sphere is published in many textbooks (*e.g.*, Potter and Foss [2], Shames [1]). The flow field, in polar (r,θ) coordinates, is given by

$$\begin{aligned} v_r &= U\cos\theta\ (1 - r_0^3/r^3) \quad \text{, and} \\ v_\theta &= -U\sin\theta\ (1 + r_0^3/r^3) \quad , \end{aligned} \tag{13}$$

where U represents the uniform free-flow velocity about a sphere of radius r_0, fixed at the origin of the coordinate system (with the flow traveling from the $-x$ toward the $+x$ direction). To make this flow unsteady, allow the free-flow velocity to be a function of time, $U(t)$. Recall, to avoid the complication of trying to force the universe to accelerate around the sphere, that the potential-flow coordinate axes, *xyz*, attached to the sphere, are in fact simultaneously traveling toward the $-x$ direction with a nonsteady velocity of magnitude $U(t)$.

Realize that this problem does not involve vorticity, employs a noninertial reference frame that does not rotate, has negligible body forces, has no shear stresses (inviscid), and is incompressible. The extended Bernoulli equation for this problem then becomes

$$\int_{R_1}^{R_2}\left(\frac{\partial V_{xyz}}{\partial t} + \frac{d^2S}{dt^2}\right)\cdot dR = -\left(\frac{V_{xyz}^2}{2} + \frac{p}{\rho}\right)\Bigg|_{R_1}^{R_2} \quad , \tag{14}$$

where R_1 and R_2 are the endpoints of the integration contour. If the integration contour is chosen to be the (straight line) stagnation contour traversing from $(-\infty,0,0)$ to $(-r_0,0,0)$, the only velocity component of relevance to the integral is the x component, so that

$$V_x(x,t) = -v_r\big|_{\theta=\pi} = U(t)\,(1 + r_0^3/x^3) \qquad (\text{for } x < -r_0,\ y = z = 0) \ . \tag{15}$$

The terms from the right-hand side of eqn (14) may thus be evaluated as follows:

$$\int_{-\infty}^{-r_0}\left(\frac{\partial V_x}{\partial t} - \frac{dU}{dt}\right)dx = -\frac{p_{stag}}{\rho} + \left(\frac{U^2}{2} + \frac{p_\infty}{\rho}\right) \ . \tag{16}$$

To finish the solution, $\partial V_x/\partial t$ needs to be evaluated from eqn (15) and substituted into eqn (16). Time-dependent potential flows (and others) often have the virtue of being separable in space and time, as in $V_x(x,t) = U(t)\cdot g(x)$. If the one-dimensional (1-D) contour length is infinite in extent, the spatial integral of the time derivative, in this separable case, may be expressed as

$$\int_{-\infty}^{-r_0}\frac{\partial V_x}{\partial t}\,dx = \frac{dU}{dt}\cdot\int_{-\infty}^{-r_0}\frac{V_x}{U}\,dx \ . \tag{17}$$

Alternately, if the 1-D contour were of finite length, eqn (17) could be expressed as

$$\int_a^b\frac{\partial V_x}{\partial t}\,dx = \frac{dU}{dt}(b-a)\cdot\frac{\int_a^b V_x\,dx}{U(b-a)} \ . \tag{18}$$

The right side of eqn (18) is composed of the free-stream acceleration multiplied by the contour length as well as a quotient factor. The quotient factor is the average velocity along the contour divided by the free-stream velocity, and, in the case of a stagnation contour, it will generally fall in the range from zero to unity, depending on the details of the flow.

In the present case, eqn (17) is utilized for the evaluation of eqn (16). It is noted that there is a canceling of the dU/dt term, which is necessary to avoid computing the force necessary to accelerate the universe. For the considered flow integrated along the specified contour, eqn (16) may be evaluated as

$$p_{stag} - p_{\infty} = \frac{\rho U^2}{2} + \frac{\rho r_0}{2} \frac{dU}{dt} \quad . \tag{19}$$

Thus, if the time-dependent velocity of the sphere is known, the stagnation pressure may be evaluated with eqn (19). That pressure varies from what would be predicted by the Bernoulli equation, eqn (1), by a term involving the acceleration rate of the sphere. If the acceleration of the sphere is positive, the stagnation pressure is seen to be higher than the Bernoulli pressure, while, if the sphere is decelerating, the stagnation pressure is less.

7. Nonsteady Solid Eroding-Rod Penetration

The problem of eroding-rod penetration has been examined by a number of researchers in recent years. The seminal works on subject were done independently by Alekseevskii [10] and Tate [11] more than 30 years ago. Tate, in subsequent work [12–13], examines the flow field associated with long-rod penetration in more detail. In the course of the work [12], the effect and magnitude of the noninertial influence are calculated for his idealized flow potential. Tate concludes that, when the long-rod penetration process can be considered as quasi-steady, the noninertial effects may be neglected. Since then, a thorough and insightful analysis of the relevant balance equations was performed by Wright and Frank [14]. Their analysis computes surface and volume integrals over the relevant region in the vicinity of the rod/target interaction zone and was able to show that the target resistance term of Alekseevskii [10] and Tate [11] encompasses more than just a simple measure of target strength.

A more recent treatise on the subject, which instead relies upon a force/momentum balance along the centerline contour only, is that of Walker and Anderson [15]. Upon assuming certain reasonable velocity fields in the tip of the rod and in the target crater, they proceed to solve the momentum equation in glorious detail, directly in the inertial XYZ laboratory frame of

reference, to include noninertial effects. The analysis presented here is not intended to supplant the esteemed work of Walker and Anderson. Rather, it is intended to show that the concepts derived herein may be very simply applied to the same problem to a similar end. Furthermore, the manner in which an accelerating coordinate system, attached to the rod/target interface, affects the overall result should be apparent in a more direct way.

In the eroding-rod problem (see Figure 2), a solid rod, of density ρ_R, instantaneous length L, and velocity V, penetrates into a semi-infinite block of density ρ_T. The rod is assumed to support a uniaxial-stress state in the longitudinal direction of the rod. The eroding interface is traveling into the target at velocity U. Furthermore, employing the assumed velocity profiles suggested by Walker and Anderson, there is a small plastically deforming region located at the eroding tip of the rod, of length s, where the velocity linearly transitions from the rigid-body rod velocity of V to the interface velocity of U. On the target side of the interface, the crater geometry (Figure 3) is locally considered a hemisphere of radius R in polar (r,θ) coordinates, with the target flow velocity, u, along the axis of symmetry decaying as

$$\frac{u}{U} = \frac{1}{\alpha^2 - 1}\left[\left(\frac{\alpha R}{r}\right)^2 - 1\right] \quad (R < r < \alpha R) \ , \tag{20}$$

while remaining zero at all distances r at and beyond αR. The parameter α defines an extent of plasticity in the target, with $\alpha > 1$ defining a finite-sized plastic zone in the target, and $\alpha \to 1^+$ denoting the limiting case of infinitesimally thin plastic zone. Along the axis of symmetry of the noninertial xyz coordinate system of Figure 3, $z = r - R$. Both the projectile and target plastic-flow zones may be considered incompressible, despite the axial velocity gradients, because of an associated radial divergence of the flow field.

Because V, U, and L are changing with time t, the reference frame attached to the rod/target interface will be a noninertial frame xyz traveling at the time-dependent velocity U. Though Walker and Anderson considered the general case of time-varying s and α, these geometry parameters are held constant for simplicity. From the perspective of the interface

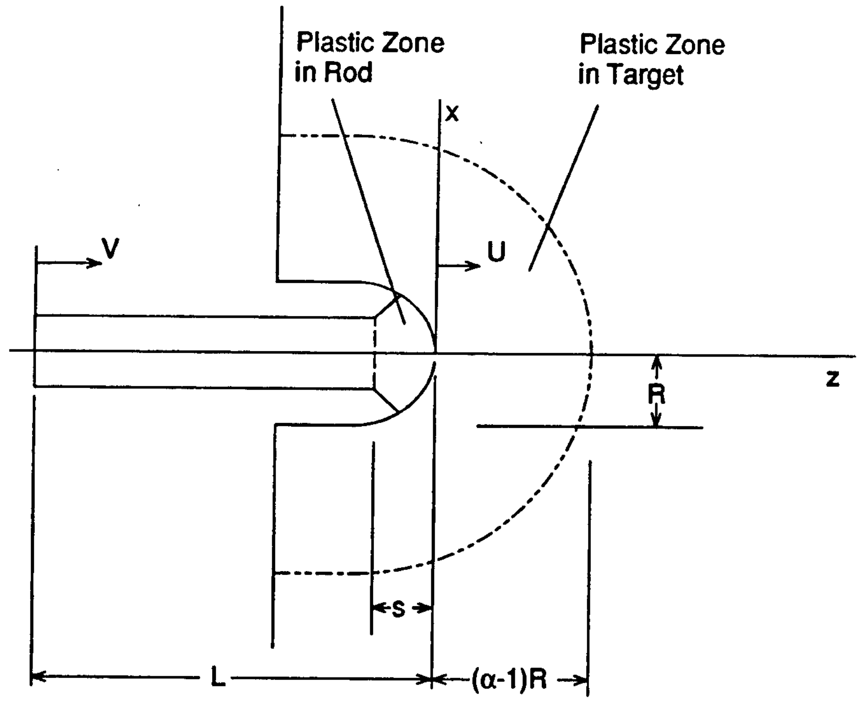

Figure 2. Geometry of the solid eroding-rod problem.

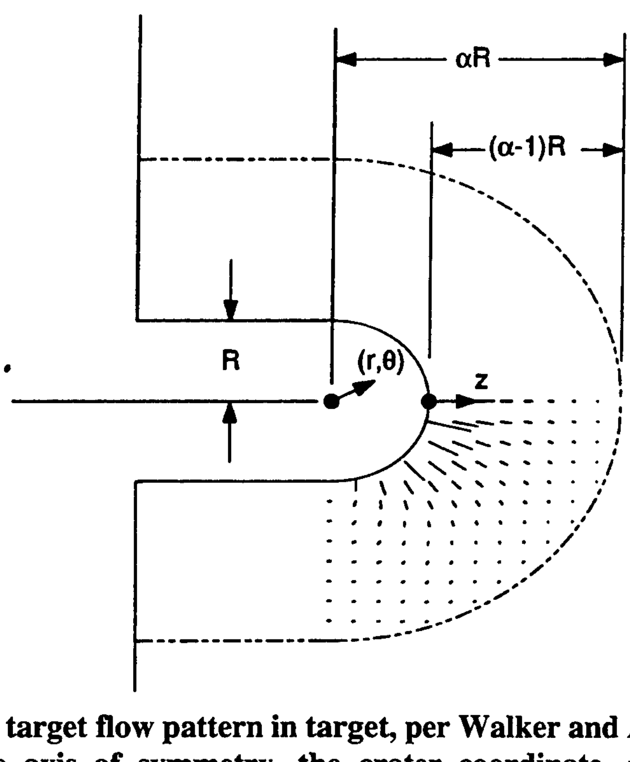

Figure 3. Assumed target flow pattern in target, per Walker and Anderson [15]. Note that, along the axis of symmetry, the crater coordinate, r, is related to interface coordinate, z, by $z = r - R$.

coordinate system xyz, the velocity as a function of axial coordinate z, along the centerline of the problem, can be given as

$$V_z = \begin{cases} V - U & -L \le z < -s \\ -(V-U)\,z/s & -s \le z < 0 \\ \dfrac{U}{\alpha^2 - 1}\left[\left(\dfrac{\alpha R}{R+z}\right)^2 - 1\right] - U & 0 \le z < (\alpha - 1)R \\ -U & z \ge (\alpha - 1)R \end{cases} \quad . \tag{21}$$

This flow field is schematically shown in Figure 4. Specifying the extended Bernoulli equation, eqn (12), along the contour defined by the axis of symmetry, for this case of irrotational (along the axis of symmetry), incompressible flow with only rectilinear accelerations, the extended Bernoulli equation reduces into a 1-D integration in z, yielding

$$\int_{z_1}^{z_2}\left(\frac{\partial V_z}{\partial t} + \frac{dU}{dt}\right)dz = -\frac{V_z^2}{2}\bigg|_{z_1}^{z_2} + \int_{z_1}^{z_2}\left(\frac{2\sigma_{xz,x} + \sigma_{zz,z}}{\rho}\right)dz \quad . \tag{22}$$

The factor of 2 on the $\sigma_{xz,x}$ term arises because of symmetry, for which $\sigma_{xz,x}$ and $\sigma_{yz,y}$ are equal on the axis of symmetry. Furthermore, because the integration contour is aligned with the z axis and the flow is incompressible, the $\sigma_{zz,z}$ integral will amount to a difference of (σ_{zz}/ρ) between the contour endpoints. This equation is, of course, identical to the momentum equation derived by Walker and Anderson, though expressed in the noninertial xyz coordinates, rather than the laboratory XYZ coordinate frame.

First, limit the fixed integration contour to the elastic portion of the solid rod, spanning the range $-L \le z < -s$, and solve eqn (22) in light of the velocity field specified by eqn (21). Because the rod velocity at $z = -s$ and $z = -L$ are identical, the contribution of the V^2 gradient integral is zero. Further, because the stress state in the rod is assumed uniaxial in z, the shear-stress-gradient integral will be exactly zero. Finally, note how the dU/dt acceleration terms from V_z and the noninertial frame acceleration cancel out. Thus, one obtains

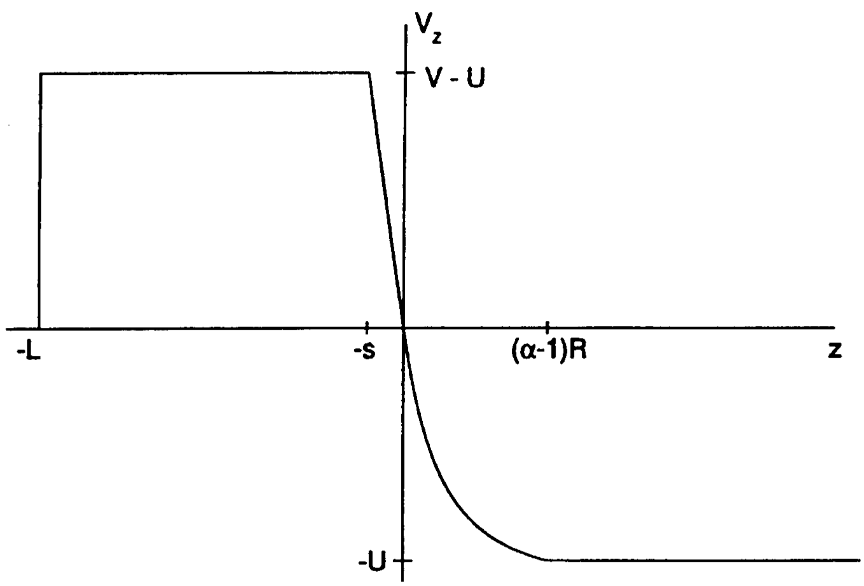

Figure 4. Schematic depicting the assumed velocity field along the axis of symmetry of the eroding rod and target.

$$\frac{dV}{dt}(L - s) = \frac{1}{\rho_R}(-Y_R - 0) \ , \tag{23}$$

where Y_R is the yield strength of the rod, being exactly the uniaxial-stress state (where tension is defined as positive) at the elastic-plastic interface [*i.e.*, $\sigma_{zz}(z = -s) = -Y_R$]. This is the high-sound-speed limiting result of Walker and Anderson (since it was not here accounted for the finite wave speed at which acceleration information travels down an elastic bar). They noted further that, were the size of the rod's plastic zone, s, a negligible percentage of the overall rod length, L, eqn (23) reduces to the Alekseevskii-Tate rod deceleration equation, $dV/dt = -Y_R/(\rho_R L)$. Despite any decelerations of the noninertial frame traveling at velocity U with the rod/target interface, the rod deceleration equation is totally independent of interface velocity U.

Secondly, reconsider eqn (22) over a different integration contour, still along the axis of symmetry but spanning from $-L \leq z < 0$, thereby including the complete rod in the integration. The terms of eqn (23) are thus retained, while adding to them the terms that arise from the small plastic zone at the tip of the eroding rod. Denoting the axial stress σ_{zz}, at the rod/target interface, as σ_{stag}, one obtains

$$\frac{dV}{dt}(L - s) + \left(\frac{dV}{dt} - \frac{dU}{dt}\right)\frac{s}{2} + \frac{dU}{dt}s = -\frac{Y_R}{\rho_R} + \frac{\sigma_{stag}}{\rho_R} - \frac{(-Y_R)}{\rho_R} + \frac{(V - U)^2}{2} \ . \tag{24}$$

This result is identical to the result of Walker and Anderson, for the case of constant plastic zone extent, s. It can be solved for the compressive stagnation stress at the rod/target interface and, by making use of a substitution of eqn (23), results in

$$-\sigma_{stag} = \frac{\rho_R (V - U)^2}{2} + Y_R - \left(\frac{dV}{dt} + \frac{dU}{dt}\right)\frac{\rho_R s}{2} \ . \tag{25}$$

If the extent of the rod's plastic zone, s, is small compared to rod length, L, or if the penetration process is steady (*i.e.*, velocity derivatives zero), the last term in eqn (25) becomes negligible and the remaining terms become identical to the expression proposed by Tate [11] for the stagnation stress on the rod side of the rod/target interface.

Turning to the target, the integration contour is defined to be along the axis of symmetry throughout the target. That is, eqn (22) is integrated between $0 \le z \le \infty$, in light of eqn (21), to obtain

$$-\frac{dU/dt}{\alpha^2-1}\left[\frac{(\alpha R)^2}{(R+z)}+z\right]_0^{(\alpha-1)R}+\left(-\frac{dU}{dt}+\frac{dU}{dt}\right)z\Big|_{(\alpha-1)R}^{\infty}$$

$$=-\frac{(-U)^2-0^2}{2}+\frac{0-\sigma_{stag}}{\rho_T}+\frac{2}{\rho_T}\int_0^{(\alpha-1)R}\frac{\partial\sigma_{xz}}{\partial x}dz+\frac{2}{\rho_T}\int_{(\alpha-1)R}^{\infty}\frac{\partial\sigma_{xz}}{\partial x}dz \quad . \tag{26}$$

The last term of both sides of the equation (the rigid-body acceleration term on the left, and the shear-stress integration on the right) are both zero, since the target material beyond the region of plastic extent, $z \ge (\alpha-1)R$, is essentially undisturbed. Solving for the axial stagnation stress on the target side of the rod/target interface gives

$$-\sigma_{stag}=\frac{\rho_T U^2}{2}+\rho_T R\frac{dU}{dt}\frac{\alpha-1}{\alpha+1}-2\int_0^{(\alpha-1)R}\frac{\partial\sigma_{xz}}{\partial x}dz \quad . \tag{27}$$

Since this paper is primarily concerned with the kinematics of nonsteady flow fields, it is not intended to delve into the constitutive relations by which the shear-stress integral along the centerline contributes to the stagnation pressure beneath an eroding rod. Walker and Anderson [15] may be consulted for these details for those interested. Suffice it to say that the terms in eqn (27) correspond identically to their terms associated with target stresses, for the special case of fixed extent of target plasticity (*i.e.*, constant α). Furthermore, they note that, for the limiting case of small crater radius, $R \to 0$ (corresponding to truly 1-D penetration), the shear-stress integral becomes the sole modification to the Bernoulli stagnation pressure. For the fixed-α case, this shear-stress integral becomes a positive constant related to the yield strength of the target material and is traditionally given the name target resistance, denoted R_T. One may infer from the result of Walker and Anderson that, in addition to the target's inertial head ($\rho_T U^2/2$), it is the target's shear-stress field, rather than the acceleration of target material under the penetrator, that is the primary contributor to interface pressure on the target side of the interface, when the penetration process is nearly steady.

By limiting the scope of problem complexity and by achieving algebraic expediency via the use of a noninertial rod/target interface coordinate system, the primary result of Walker and Anderson, who spent quite a number of journal pages exhaustively addressing this subject, has been recreated in the span of several paragraphs. For those who don't wish to dwell on the solid-mechanics aspects of their derivations, the parts of the problem dealing with accelerations and noninertial frames can be grasped here, in their essence.

8. Consequences of Noninertiality

The means of accounting for the noninertiality of a reference system have long been established and are embodied in eqn (9). Failure to properly take these terms into account, however, will lead to erroneous calculations in various forms. Consider the two example problems examined in the preceeding text to see the consequences of improperly applying the momentum equation in a noninertial frame.

For the accelerating sphere problem, failing to subtract out the dU/dt acceleration of the noninertial xyz frame would have added a term to the stagnation pressure, eqn (19), of magnitude $\rho \cdot dU/dt$ multiplied by the contour length, call it l. Obviously, for a contour of infinite length, the error would be infinite, resulting from the fact that the pressure being computed arose from accelerating the whole mass of the universe about the sphere. If the contour length were finite, the added pressure term, being proportional to contour length, is like the situation existing within a (inviscid) wind tunnel. Additional pressure head needs to be supplied to the tunnel in order to accelerate the flow through the tunnel test section. A quick inspection of the form of eqn (19) (augmented on the right side by $\rho\, l \cdot dU/dt$) reveals that, as the pressure differential is raised across the test section, the flow velocity will accelerate to eventually reach a new equilibrium velocity. The length of the test-section contour, l, denoting the length (and thus mass) of the flow to be accelerated, will govern the time constant of the acceleration. So, in the case of the problem of an accelerating sphere, improperly ignoring the noninertiality of the reference frame actually changes the problem to one of a sphere fixed in a wind tunnel.

For the eroding-rod problem, the consequences of ignoring the noninertiality of the interface coordinate system produce a different set of errors. The consequences upon the rod deceleration equation, eqn (23), would literally be to replace dV/dt with $d(V - U)/dt$, as in

$$\frac{d(V - U)}{dt}(L - s) = -\frac{Y_R}{\rho_R} \,. \tag{28}$$

For a symmetric impact of like materials at speeds above the elastic limit, U will typically be on the order of $V/2$. In such a case, the effect on rod deceleration will be an error on the order of 100%. For cases where U is a larger percentage of V, as in the case of high-density-rod penetration, the error in the deceleration calculation is correspondingly increased.

On the target side of the interface, failure to account for the acceleration of the coordinate system will introduce a $-\rho_T \cdot l \cdot dU/dt$ contribution to right side of the momentum balance equation, eqn (27), as in

$$-\sigma_{stag} = \frac{\rho_T U^2}{2} + \rho_T R \frac{dU}{dt}\frac{\alpha - 1}{\alpha + 1} - 2\int_0^{(\alpha-1)R} \frac{\partial \sigma_{xz}}{\partial x}\, dz - \rho_T l \frac{dU}{dt} \,. \tag{29}$$

Here, l denotes a contour length of target material to be integrated [assumed greater than or equal to $(\alpha - 1) R$], and dU/dt is negative for a decelerating rod. The first warning flag is that the last term of eqn (29) is proportional to the contour length which, for a semi-infinite target, is infinite in length. Such an improper inertial interpretation again leads to a calculation of the force to accelerate the universe with respect to the rod/target interface. If, on the other hand, the thickness of the target were finite, or if the integrated contour length, l, were arbitrarily made finite, eqn (29) though quite incorrect, might seem less obviously so. If length l of the integrated contour were large enough to dominate the other terms of eqn (29), leading to

$$\sigma_{stag} \approx \rho_T l \frac{dU}{dt} \,, \tag{30}$$

one might erroneously conclude that the normal interface stress, σ_{stag}, is primarily supported by the ''apparent'' deceleration of target material relative to the rod/target interface, rather than by

the inertial head and elastic shear-stress distribution within the target. In reality, the true effect of interface deceleration [second term on the right side of eqns (27) and (29)] has just the opposite effect (*i.e.*, opposite sign): when the interface and associated target material are traveling at velocity U, a deceleration of the interface actually **lowers** the stagnation stress because not only is U made lower in the process, but also the associated target material is inertially tending to travel at U and resists any decrease in interface velocity. This resistance of target material to decelerate (*i.e.*, the inertia of the plastically entrained crater material) would have the effect of superimposing an axial-tension field on top of the steady-state (inertial) compression distribution. Thus, the act of interface deceleration actually lowers the interface stress.

Another reality check, which would indicate the inappropriateness of eqn (30), is the inference that a positive acceleration of the rod/target interface would be met with tension at the interface. Such accelerations invariably occur, when the penetration of a multilayered target transitions from a high-density target element to a lower density element of comparable strength. Yet, it is known that such a transition is not accompanied by tension at the rod/target interface. Thus, in the case of an eroding rod undergoing deceleration, it may be concluded that a proper accounting of the noninertial behavior of the rod/target interface is crucial to a proper formulation of the overall problem.

9. Conclusions

Once the groundwork for the extended Bernoulli equation, eqn (12), has been laid, the solution to actual problems can often proceed quickly. All of the concepts necessary to develop this equation have existed in the educational literature of fluid and solid mechanics and dynamics for many years, if not centuries. However, all of the applicable terms contributing to the equation are not generally located in a single source, as the educational literature prefers to expeditiously reduce the governing momentum equation to special-case solutions for instructional purposes. These special-case limitations often include steady, incompressible, irrotational, or inviscid flows in inertial reference frames.

The momentum equation along an integration contour within a general flow field has been herein rederived. By placing no restrictions on the type or manner of flow, the equation has been presented, using the popular terminology, as an ''extended'' Bernoulli equation. The equation, as presented, is valid for unsteady, compressible, rotational, elasto-viscoplastic flows measured relative to a noninertial (translating and/or rotating) coordinate system, whose motion is known.

Of particular interest were flows measured relative to noninertially translating coordinate systems. As such, two example problems of this variety have been solved in this report. The effect of coordinate system noninertiality introduces additional terms into the momentum equation, which are only ignored at the peril of the investigator. In the case of a rigid sphere accelerating within a quiescent inviscid medium, a failure to consider the noninertial terms has the effect of solving a different, though valid, problem of a stationary sphere in a wind tunnel.

In the case of the solid eroding-rod problem, by comparing the present analysis to that of Walker and Anderson [15] (who solved the identical problem in the inertial laboratory frame of reference), it was observed that choosing a convenient coordinate system (even if noninertial) can significantly simplify the algebraic manipulation of the governing momentum equation. However, if misapplied, the consequence of failing to account for the acceleration of the eroding interface produces significant errors, numerically and conceptually. First, the rod deceleration rate is miscalculated, often by a factor of 2 or greater. Also, in the target, the basic understanding of the problem is completely distorted by failure to properly account for the noninertiality of the interface reference frame. In reality, the inertial head and elastic shear-stress distribution within the target are primarily responsible for the buildup of interface pressure, while the interface deceleration, because of the target-material inertia in the plastically entrained zone of the target, actually ameliorates the interface stress. From the point of view of the noninertial frame however, one might erroneously conclude that the interface deceleration was actually the primary cause for the buildup of stress on the target side of the interface—a conclusion totally opposite from and in contradiction to the properly formulated (inertial) momentum balance.

This report presents a general form of the momentum equation that is extremely useful for solving a great variety of problems that might not otherwise fall into idealized categories. The solved examples help to illustrate the power of choosing a convenient frame of reference in which to solve a given problem. However, the examples also serve to emphasize the vital importance of properly accounting for effects of accelerating reference frames.

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13. ABSTRACT ***(Maximum 200 words)***

A general form of the momentum equation is presented. Because the solution is presented as an integral along a flow line, it is here referred to as an "extended" Bernoulli equation. The equation, as presented, is valid for unsteady, compressible, rotational, elasto-viscoplastic flows measured relative to a noninertial (translationally and/or rotationally accelerating) coordinate system, whose motion is known. Though all of these concepts have long been separately addressed in the educational literature of fluid and solid mechanics and dynamics, they are usually not available from a single source, as the literature prefers to reduce the problem to special-case solutions for instructional purposes. Two examples that make use of the extended Bernoulli equation in noninertial reference frames are solved. The consequences of failing to properly account for noninertial effects are discussed.

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Bernoulli, noninertial, streamlines, penetration, acceleration

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